Perfect Quantum Teleportation and Superdense coding with $P_{max} = 1/2$ states

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Abstract

We conjecture that criterion for perfect quantum teleportation is that the Groverian entanglement of the entanglement resource is $1/\sqrt{2}$. In order to examine the validity of our conjecture we analyze the quantum teleportation and superdense coding with $|\Phi\rangle = (1/\sqrt{2})(|00q_1\rangle + |11q_2\rangle)$, where $|q_1\rangle$ and $|q_2\rangle$ are arbitrary normalized single qubit states. It is shown explicitly that $|\Phi\rangle$ allows perfect two-party quantum teleportation and superdense coding scenario. Next we compute the Groverian measures for $|\psi\rangle = \sqrt{1/2 - b^2}|100\rangle + b|010\rangle + a|001\rangle + \sqrt{1/2 - a^2}|111\rangle$ and $|\tilde{\psi}\rangle = a|000\rangle + b|010\rangle + \sqrt{1/2 - (a^2 + b^2)}|100\rangle + (1/\sqrt{2})|111\rangle$, which also allow the perfect quantum teleportation. It is shown that both states have $1/\sqrt{2}$ Groverian entanglement measure, which strongly supports that our conjecture is valid.

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Quantum teleportation[1] is a physical process, where an unknown state can be transmitted from one remote place to another by making use of the entanglement resource and classical communication. About one and half decades ago Bennett et al[1] have found such process. They used the two-qubit Einstein-Podolsky-Rosen (EPR) state as an entanglement resource, which is assumed to be initially shared between the sender, called Alice and the receiver, called Bob. Quantum teleportation is generalized to the case where the noisy channels make a quantum channel to be mixed state[2]. In this case the quantum teleportation generally becomes imperfect due to the effect of the noisy channels. If we do not have EPR state or its local-unitary(LU) equivalents, then Alice cannot teleport a single qubit to Bob with unit fidelity and unit probability. Increasing the fidelity as much as possible, one can achieve an unit fidelity with a probability less than a unit, which is called probabilistic quantum teleportation[3, 4].

The higher-qubit entangled states also can be used as an entanglement resource of the quantum teleportation. In n-qubit system with $n \geq 3$ there seems to be no unique way to define the maximally entangled states. For $n = 3$, for example, it is well-known that there are two types of entangled states called Greenberger-Horne-Zeilinger(GHZ) state[5]

$$|GHZ\rangle = \frac{1}{\sqrt{2}} \left(|000\rangle + |111\rangle \right) \tag{1}$$

and W state[6]

$$|W\rangle = \frac{1}{\sqrt{3}} \left(|001\rangle + |010\rangle + |100\rangle \right). \tag{2}$$

These two types are not connected to each other via stochastic local operations and classical communication(SLOCC)[6]. It was also found that four qubits can be entangled in nine different ways[7].

We generally use the entanglement measures to quantify the entanglement of multi-qubit state $|\psi\rangle$. One of the well-known measure constructed by an operational method[1] is a Groverian measure[8] defined $G(\psi) \equiv \sqrt{1 - P_{max}}$, where

$$P_{max} = \max_{|e_1\rangle \cdots |e_n\rangle} |\langle e_1| \otimes \cdots \otimes \langle e_n|\psi\rangle|^2. \tag{3}$$

[1] In operational method the entanglement measures are constructed by making use of the real physical tasks such as quantum algorithms. In fact, the Groverian entanglement measure was constructed from Grover's search algorithm.

Physically, P_{max} corresponds to the maximal probability of success in Grover's search algorithm[9] when $|\psi\rangle$ is n-qubit initial state. Eq.(3) can be re-written in terms of density matrix $\rho = |\psi\rangle\langle\psi|$ in the form

$$P_{max} = \max_{R^1 \cdots R^n} \text{Tr}\left[\rho R^1 \otimes \cdots \otimes R^n\right] \tag{4}$$

where $R^i \equiv |q_i\rangle\langle q_i|$.

The quantum teleportation with 3-qubit GHZ state was discussed in Ref.[10]. When one sender (Alice) would like to send one-qubit state

$$|\tilde{\psi}\rangle_1 = \alpha|0\rangle + \beta|1\rangle \qquad\qquad (|\alpha|^2 + |\beta|^2 = 1) \tag{5}$$

to one receiver (Bob), the perfect quantum teleportation with $|GHZ\rangle_{234}$ can be easily shown as following. First, we assume that Alice has particles 2 and 3, and Bob has particle 4. Next, we note that $|\tilde{\psi}\rangle_1 \otimes |GHZ\rangle_{234}$ reduces to

$$\begin{aligned} |\tilde{\psi}\rangle_1 \otimes |GHZ\rangle_{234} = \Big[& \sqrt{P_1^+}|\phi_1^+\rangle_{123} \otimes \mathbb{1} + \sqrt{P_1^-}|\phi_1^-\rangle_{123} \otimes Z \\ & + \sqrt{P_2^+}|\phi_2^+\rangle_{123} \otimes X + \sqrt{P_2^-}|\phi_2^-\rangle_{123} \otimes ZX \Big] (\alpha|0\rangle_4 + \beta|1\rangle_4) \end{aligned} \tag{6}$$

where $P_1^+ = P_1^- = P_2^+ = P_2^- = 1/4$, (X, Y, Z) Pauli matrices, and

$$\begin{aligned} |\phi_1^\pm\rangle &= \frac{1}{\sqrt{2}}(|000\rangle \pm |111\rangle) \\ |\phi_2^\pm\rangle &= \frac{1}{\sqrt{2}}(|100\rangle \pm |011\rangle). \end{aligned} \tag{7}$$

Since $|\phi_1^\pm\rangle$ and $|\phi_2^\pm\rangle$ are orthogonal to each others, Alice can distinguish them via von Neumann type measurement. Of course, the postulates of quantum mechanics tells that the probabilities for outcomes are $P_1^\pm$ and $P_2^\pm$, respectively. After Alice conveys her measurement results to Bob via classical channel, Bob can construct $|\tilde{\psi}\rangle$ by applying an appropriate unitary transformation to his own qubit. This is a whole story of quantum teleportation between two parties.

Since the quantum teleportation between two parties can be done perfectly with the two-qubit EPR channel, actually the above-mentioned teleportation is not new scheme. However, the three-qubit GHZ state can be used to three-party (Alice, Bob, Cliff) teleportation. Although the well-known no-cloning theorem[11, 12] does not allow for Alice to teleport $|\tilde{\psi}\rangle$ to both Bob and Cliff, one can use the 3-qubit GHZ state as a quantum copier (cloning device)[13–15] with fidelity less than one[10]. The quantum teleportation with four qubit GHZ state and its role as a cloning machine was discussed in Ref.[16].

Recently, furthermore, the slightly-modified W state

$$|W_1\rangle_{234} = \frac{1}{2}\left(|100\rangle_{234} + |010\rangle_{234} + \sqrt{2}|001\rangle_{234}\right) \tag{8}$$

is used for perfect two-party quantum teleportation[17]. This can be shown as following. First let us assume that Alice has particles 2 and 3, and Bob has particle 4. Then after some calculation it is easy to show

$$\begin{aligned}|\tilde{\psi}\rangle_1 \otimes |W_1\rangle_{234} = \sqrt{\frac{1}{4}}\Big[&|\eta_1^+\rangle_{123} \otimes \mathbb{1} + |\eta_1^-\rangle_{123} \otimes Z \\ &+|\xi_1^+\rangle_{123} \otimes X + |\xi_1^-\rangle_{123} \otimes ZX\Big](\alpha|0\rangle_4 + \beta|1\rangle_4)\end{aligned} \tag{9}$$

where

$$\begin{aligned}|\eta_1^\pm\rangle &= \frac{1}{2}\left(|010\rangle + |001\rangle \pm \sqrt{2}|100\rangle\right) \\ |\xi_1^\pm\rangle &= \frac{1}{2}\left(|110\rangle + |101\rangle \pm \sqrt{2}|000\rangle\right).\end{aligned} \tag{10}$$

Since $|\eta_1^\pm\rangle$ and $|\xi_1^\pm\rangle$ are orthogonal to each others, the usual quantum teleportation process allows Bob to have $|\tilde{\psi}\rangle$ via an appropriate unitary transformation. The difference of this process from teleportation with $|GHZ\rangle$ is that in this case Alice should initially choose particles 2 and 3 for perfect teleportation. If Alice has different particles, one can show that the perfect teleportation is impossible with state $|W_1\rangle$. Since, however, initially Alice and Bob can choose particles freely, we can use $|W_1\rangle$ for perfect two-party quantum teleportation.

The perfect teleportation with $|W_1\rangle$ and $|GHZ\rangle$ naturally arises a question: what is a criterion for the perfect two-party quantum teleportation? In other words what common property of $|W_1\rangle$ and $|GHZ\rangle$ allows perfect teleportation? As will be shown below, $|W_1\rangle$ and $|GHZ\rangle$ have same $P_{max} = 1/2$. Thus this fact might be criterion for the perfect teleportation. The purpose of this paper is to explore this issue in detail.

Recently, it was shown[18] that P_{max} for n-qubit state can be computed if one knows one of the $(n-1)$-qubit reduced states using a formula

$$P_{max} = \max_{R^1\cdots R^n} \mathrm{Tr}\left[\rho R^1 \otimes \cdots \otimes R^n\right] = \max_{R^1\cdots R^{n-1}} \mathrm{Tr}\left[\rho R^1 \otimes \cdots \otimes R^{n-1} \otimes \mathbb{1}\right]. \tag{11}$$

Eq.(11) leads several important conclusions[18]. Furthermore, Eq.(11) provides a good tool for the analytic calculation of P_{max}. In Ref.[19] P_{max} for various 3-qubit states was analytically computed using Eq.(11). For the generalized W state, for example,

$$|GW\rangle = a|001\rangle + b|010\rangle + c|100\rangle \qquad\qquad (a^2 + b^2 + c^2 = 1) \tag{12}$$

P_{max} can be expressed as following:

$$P_{max} = \begin{cases} \max(a^2, b^2, c^2) = \alpha^2 & \text{when } \alpha^2 \geq \beta^2 + \gamma^2 \\ 4R^2 & \text{when } \alpha^2 \leq \beta^2 + \gamma^2 \end{cases} \tag{13}$$

where $\alpha^2 = \max(a^2, b^2, c^2)$ and, β^2 and γ^2 are the remaining ones. In Eq.(13) R is a circumradius of the triangle a, b, c. From Eq.(13) it is easy to show that if a, b, c form an equilateral triangle, $P_{max} = 4/9$, which is consistent with the results of Ref.[20]. Furthermore, Eq.(13) implies that if the parameters a, b, c form a right triangle, we call the corresponding $|GW\rangle$ 'singular states'[2] and their P_{max} becomes $1/2$. Since $|W_1\rangle$ in Eq.(8) is one of singular states, its P_{max} is $1/2$ as we commented before. Since it is well-known that the n-qubit GHZ states has $P_{max} = 1/2$ regardless of n[20], this remarkable fact makes us conjecture that $P_{max} = 1/2$ is a necessary (or sufficient) condition for the perfect two-party teleportation.

To examine the validity of our conjecture we choose the state

$$|\Phi\rangle_{234} = \frac{1}{\sqrt{2}} (|00q_1\rangle + |11q_2\rangle) \tag{14}$$

where $|q_1\rangle$ and $|q_2\rangle$ are arbitrary normalized one-qubit states. If $|q_1\rangle = |0\rangle$ and $|q_2\rangle = |1\rangle$, $|\Phi\rangle$, of course, becomes usual GHZ state. As shown in Ref.[19], P_{max} of $|\Phi\rangle$ is also $1/2$ regardless of $|q_1\rangle$ and $|q_2\rangle$. Thus this state is appropriate to check the validity of our conjecture. We will show that like GHZ and W states $|\Phi\rangle$ also allows the perfect quantum teleportation and superdense coding scenario. Next, we will compute P_{max} of more general three-qubit states,

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[2] The suitability of the terminology 'singular states' can be seen easily if one changes Eq.(12) into the one-parameter dependent states by letting $b = \kappa a$ and $c = \kappa^2 a$. Then Eq.(13) allows oneself to derive the analytic expressions for the κ-dependence of P_{max}. Using these expressions, one can show that P_{max} at right triangle a, b, c is continuous but its derivative $dP_{max}/d\kappa$ is discontinuous. For general state (12) the normalization condition $a^2 + b^2 + c^2 = 1$ defines a sphere and the condition for right triangle $\alpha^2 = \beta^2 + \gamma^2$ defines a cone. The intersection of the cone with the sphere is generally a circle on the sphere. Inside the circle $P_{max} \leq 1/2$ and outside $P_{max} \geq 1/2$. On the circle $P_{max} = 1/2$ but its gradient is discontinuous. Thus it is reasonable to use the terminology 'singular states' for the states with $\alpha^2 = \beta^2 + \gamma^2$.

which also allow the perfect teleportation. As we conjecture, it is shown that these general states also have $P_{max} = 1/2$.

In order to discuss the two-party quantum teleportation with $|\Phi\rangle$ we assume first that Alice has particles 3 and 4, and Bob has particle 2. In this situation it is convenient to define

$$|\psi_1^{\pm}\rangle = \frac{1}{\sqrt{2}}\left[|00q_1\rangle \pm |11q_2\rangle\right] \tag{15}$$
$$|\psi_2^{\pm}\rangle = \frac{1}{\sqrt{2}}\left[|10q_1\rangle \pm |01q_2\rangle\right].$$

Then one can show that $|\psi_1^{\pm}\rangle$ and $|\psi_2^{\pm}\rangle$ are orthogonal to each others regardless of $|q_1\rangle$ and $|q_2\rangle$. After some calculation one can show straightforwardly that $|\tilde{\psi}\rangle_1 \otimes |\Phi\rangle_{234}$ reduces to

$$|\tilde{\psi}\rangle_1 \otimes |\Phi\rangle_{234} = \sqrt{\frac{1}{4}}\Big[|\psi_1^{+}\rangle_{134} \otimes \mathbb{1} + |\psi_1^{-}\rangle_{134} \otimes Z \tag{16}$$
$$+|\psi_2^{+}\rangle_{134} \otimes X + |\psi_2^{-}\rangle_{134} \otimes ZX\Big](\alpha|0\rangle_2 + \beta|1\rangle_2).$$

Thus Alice can send $|\tilde{\psi}\rangle$ to Bob via usual quantum teleportation process: she distinguishes $|\psi_1^{\pm}\rangle$ and $|\psi_2^{\pm}\rangle$ via von Neumann type measurement and conveys her measurement outcomes to Bob via classical channel. If Bob has particle 3 and Alice has particles 2 and 4, one can show similarly that a perfect quantum teleportation with $|\Phi\rangle$ is also possible.

Finally, let us consider the situation that Bob has particle 4 and Alice has particles 2 and 3. Even in this case one can show that perfect quantum teleportation is possible if $|q_1\rangle$ is orthogonal to $|q_2\rangle$, *i.e.* $\langle q_1|q_2\rangle = 0$. If $\langle q_1|q_2\rangle = 0$, there should exist an unitary operator u such that $|q_1\rangle = u|0\rangle$ and $|q_2\rangle = u|1\rangle$ because unitary operator preserves the inner product. Then $|\Phi\rangle$ is obtained from $|GHZ\rangle$ via local-unitary transformation as following

$$|\Phi\rangle = (\mathbb{1} \otimes \mathbb{1} \otimes u)\,|GHZ\rangle. \tag{17}$$

Then Eq.(6) implies

$$|\tilde{\psi}\rangle_1 \otimes |\Phi\rangle_{234} = \sqrt{\frac{1}{4}}\Big[|\phi_1^{+}\rangle_{123} \otimes u\mathbb{1} + |\phi_1^{-}\rangle_{123} \otimes uZ \tag{18}$$
$$+|\phi_2^{+}\rangle_{123} \otimes uX + |\phi_2^{-}\rangle_{123} \otimes uZX\Big]\,(\alpha|0\rangle_4 + \beta|1\rangle_4).$$

Therefore, if Alice has outcome $|\phi_1^{+}\rangle$ via her measurement, Bob can get $|\tilde{\psi}\rangle$ by operating $u^{-1} = u^{\dagger}$ to his qubit. If she has $|\phi_1^{-}\rangle_{123}$, $|\phi_2^{+}\rangle_{123}$ and $|\phi_2^{-}\rangle_{123}$ respectively, Bob should operate Zu^{-1}, Xu^{-1} and XZu^{-1} for each case to get $|\tilde{\psi}\rangle$. Thus perfect quantum teleportation is possible. Although perfect two-party quantum teleportation is impossible provided that Bob has initially particle 4 and $\langle q_1|q_2\rangle \neq 0$, we can use $|\Phi\rangle$ for perfect teleportation because initially Alice and Bob can choose their particles freely. This is exactly same situation with teleportation with $|W_1\rangle$. In conclusion we can use $|\Phi\rangle$ for the perfect two-party quantum teleportation. This strongly supports our conjecture that *the criterion for the perfect quantum teleportation is* $P_{max} = 1/2$.

Next we would like to discuss the superdense coding[21] with $|\Phi\rangle$. In order for the superdense coding scenario to work Alice should be able to send two classical bits to Bob by sending one qubit. Now we assume that Alice has particle 2 and Bob has particles 3 and 4 in $|\Phi\rangle_{234}$. If Alice applies $(\mathbb{1}, Z, X, -iY)$ to her qubit, $|\Phi\rangle$ changes into

$$(\mathbb{1} \otimes \mathbb{1} \otimes \mathbb{1})\,|\Phi\rangle = |\psi_1^+\rangle \tag{19}$$
$$(Z \otimes \mathbb{1} \otimes \mathbb{1})\,|\Phi\rangle = |\psi_1^-\rangle$$
$$(X \otimes \mathbb{1} \otimes \mathbb{1})\,|\Phi\rangle = |\psi_2^+\rangle$$
$$(-iY \otimes \mathbb{1} \otimes \mathbb{1})\,|\Phi\rangle = |\psi_2^-\rangle$$

respectively. Since $|\psi_1^{\pm}\rangle$ and $|\psi_2^{\pm}\rangle$ are orthogonal to each other, Bob can distinguish them via von Neumann type measurement if Alice send her one qubit to him. This completes the superdense coding scenario with $|\Phi\rangle$. Similarly, one can show that we can complete the superdense coding scenario if Alice has particle 3 and Bob has particles 2 and 4. Although perfect superdense coding scenario is impossible provided that Alice has particle 4 and $\langle q_1|q_2\rangle \neq 0$, we can use $|\Phi\rangle$ for prefect superdense coding because initially Alice and Bob can share particles freely at their convenience.

Recently, two three-qubit states were found, which allow the perfect quantum teleportation[22]. We would like to show that both states also have $P_{max} = 1/2$ as we conjecture. This strongly supports the validity of our conjecture again. First state is

$$|\psi\rangle = \sqrt{\frac{1}{2} - b^2}|100\rangle + b|010\rangle + a|001\rangle + \sqrt{\frac{1}{2} - a^2}|111\rangle. \qquad \left(0 \leq a, b \leq \frac{1}{\sqrt{2}}\right) \tag{20}$$

If $a = 1/\sqrt{2}$ and $b = 1/2$, $|\psi\rangle$ reduces to $|W_1\rangle$ defined in Eq.(8). Let $\{\alpha, \beta, \gamma, \delta\}$ be set of $\{a, b, \sqrt{1/2 - a^2}, \sqrt{1/2 - b^2}\}$ with decreasing order. Then one can show easily $\alpha^2 \leq \beta^2 + \gamma^2 + \delta^2 + 2\beta\gamma\delta/\alpha$ regardless of a and b. As shown in Ref.[23], then, P_{max} for $|\psi\rangle$ equals to $4R^2$, where R is a circumradius of convex quadrangle:

$$R^2 = \frac{(a_1 a_2 + a_3 a_4)(a_1 a_3 + a_2 a_4)(a_1 a_4 + a_2 a_3)}{4\omega^2 - r_3^2} \tag{21}$$

where $\omega = a_1 a_2 + a_3 a_4$, $r_3 = a_1^2 + a_2^2 - a_3^2 - a_4^2$, and the constants a_i's are the coefficients of the quantum channel (20). If we put $a_1 = \sqrt{1/2 - b^2}$, $a_2 = b$, $a_3 = a$ and $a_4 = \sqrt{1/2 - a^2}$, one can show easily that P_{max} for $|\psi\rangle$ in Eq.(20) is 1/2.

Second state which allows a perfect quantum teleportation is

$$|\tilde{\psi}\rangle = a|000\rangle + b|010\rangle + \sqrt{\frac{1}{2} - (a^2 + b^2)}|100\rangle + \frac{1}{\sqrt{2}}|111\rangle. \qquad \left((0 \leq a^2 + b^2 \leq \frac{1}{2}\right) \tag{22}$$

If $a = 1/\sqrt{2}$ and $b = 0$, $|\tilde{\psi}\rangle$ exactly coincides with GHZ state. As shown in Ref.[18, 19], P_{max} for $|\tilde{\psi}\rangle$ can be written as

$$P_{max} = \max_{|\vec{s}_2|=|\vec{s}_3|=1} \frac{1}{4} \left[1 + \vec{s}_2 \cdot \vec{r}_2 + \vec{s}_3 \cdot \vec{r}_3 + s_{2i} s_{3j} g_{ij}\right] \tag{23}$$

where

$$\begin{aligned}
&\vec{r}_2 = \mathrm{Tr}[\rho^B \vec{\sigma}] = (2ab, 0, -2b^2) \qquad (24)\\
&\vec{r}_3 = \mathrm{Tr}[\rho^C \vec{\sigma}] = (0, 0, 0)\\
&g_{ij} = \mathrm{Tr}[\rho^{BC} \sigma_i \otimes \sigma_j] = \begin{pmatrix} \sqrt{1 - 2(a^2 + b^2)} & 0 & 2ab \\ 0 & -\sqrt{1 - 2(a^2 + b^2)} & 0 \\ 0 & 0 & 1 - 2b^2 \end{pmatrix}.
\end{aligned}$$

In Eq.(24) ρ^{BC}, ρ^B and ρ^C are the corresponding partial traces of $\rho^{ABC} \equiv |\tilde{\psi}\rangle\langle\tilde{\psi}|$ and σ_i's are usual Pauli matrix. Due to maximization in Eq.(23), $\vec{s}_2$ and $\vec{s}_3$ satisfy the Lagrange multiplier equations

$$\begin{aligned}
&\vec{r}_2 + g\vec{s}_3 = \Lambda_1 \vec{s}_2 \qquad (25)\\
&\vec{r}_3 + g^T \vec{s}_2 = \Lambda_2 \vec{s}_3
\end{aligned}$$

with $\Lambda_1, \Lambda_2 > 0$. Let $\vec{s}_2 = (s_{2x}, s_{2y}, s_{2z})$ and $\vec{s}_3 = (s_{3x}, s_{3y}, s_{3z})$. Then Lagrange multiplier equations in general reduce to six-degree algebraic equations and it is usually impossible to derive the solutions analytically. For $|\tilde{\psi}\rangle$, however, Eq.(25) reduce to simple cubic equations due to dramatic cancellation between left and right sides. Due to this cancellation we can derive analytic solutions which are $s_{2y} = s_{3y} = 0$ and

$$s_{3z} = \frac{(a^2 - b^2) + 2b^2(a^2 + b^2)}{(a^2 + b^2)(1 - 2b^2)} \qquad \text{or} \qquad \pm b\sqrt{\frac{1 - 2(a^2 + b^2)}{(a^2 - b^2)(1 - 2b^2)}}. \tag{26}$$

The first solution of Eq.(26) yields the remaining solutions

$$s_{3x} = \frac{2ab\sqrt{1-2(a^2+b^2)}}{(a^2+b^2)(1-2b^2)} \qquad s_{2x} = \frac{2ab}{a^2+b^2} \qquad s_{2z} = \frac{a^2-b^2}{a^2+b^2} \tag{27}$$

and positive Lagrange multiplier constants $\Lambda_1 = 1$ and $\Lambda_2 = 1 - 2b^2$. Then Eq.(23) gives $P_{max} = 1/2$. The second solution of Eq.(26) also yields the different remaining solutions, but the corresponding P_{max} is $(1/4)(1+\sqrt{1-2a^2})$, which is smaller than $1/2$. Since we should take maximization in Eq.(23), P_{max} for $|\tilde{\psi}\rangle$ should be $1/2$.

We have shown that $|\Phi\rangle$, whose P_{max} is $1/2$, allows the perfect two-party quantum teleportation. Also we have shown that $|\psi\rangle$ and $|\tilde{\psi}\rangle$ in Eq.(20) and Eq.(22) have $P_{max} = 1/2$. The usual GHZ and W states are special limits of $|\psi\rangle$ and $|\tilde{\psi}\rangle$. This means that our conjecture "*the criterion for the perfect two-party quantum teleportation is* $P_{max} = 1/2$" is widely applicable. Since we cannot find any counter-example, we feel that this criterion is a necessary and sufficient condition. In other words "*the perfect two-party quantum teleportation is possible if and only if the Groverian measure for the entanglement resource is* $1/\sqrt{2}$". But more rigorous proof is needed for this statement.

If our conjecture is right, it can be used to find the quantum states which allow the perfect teleportation. For example, let us consider the four-qubit states. Unfortunately, we do not know how to compute the Groverian measure of the general four-qubit states analytically until now except very rare cases. Since, however, the techniques for the analytic computation of P_{max} are developed rapidly, we believe that we have many formula about Groverian measures for the four-qubit states in the near future. Then our conjecture can be applied to find the optimal states for the perfect teleportation or quantum copier (cloning device). In addition, at least, we know how to compute the Gorverian measure numerically[20]. The numerical calculation gives $0.499 \leq P_{max} \leq 0.5$ for

$$|\psi\rangle = a|1000\rangle + b|0100\rangle + c|0010\rangle + d|0001\rangle \tag{28}$$

when $\alpha^2 = \beta^2 + \gamma^2 + \delta^2$ where $\{\alpha, \beta, \gamma, \delta\}$ is $\{a, b, c, d\}$ with decreasing order. Thus if our conjecture is correct, $|\psi\rangle$ is Eq.(28) may allow the perfect or imperfect (with very high fidelity) teleportation. Thus the rigorous proof for our conjecture is important in this context. We hope to keep on studying toward the application of our conjecture and its complete proof.

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references

[1] C. H. Bennett, G. Brassard, C. Crépeau, R. Jozsa, A. Peres and W. K. Wootters, *Teleporting an Unknown Quantum State via Dual Classical and Einstein-Podolsky-Rosen Channles*, Phys. Rev. Lett. **70** (1993) 1895.

[2] S. Oh, S. Lee and H. Lee, *Fidelity of quantum teleportation through noisy channels*, Phys. Rev. **A66** (2002) 022316 [quant-ph/0206173].

[3] P. Agrawal and A. K. Pati, *Probabilistic quantum teleportation*, Phys. Lett. **A305** (2002) 12 [quant-ph/0210004].

[4] G. Gordon and G. Rigolin, *Generalized teleportation protocol*, Phys. Rev. **A73** (2006) 042309 [quant-ph/0511077].

[5] D. M. Greenberger, M. Horne, and A. Zeilinger, *Bell's Theorem, Quantum Theory, and Conceptions of the Universe*, edited by M. Kafatos (Kluwer, Dordrecht, 1989) p 69.

[6] W. Dür, G. Vidal, and J. I. Cirac, *Three qubits can be entangled in two inequivalent ways*, Phys. Rev. **A62** (2000) 062314 [quant-ph/0005115].

[7] F. Verstraete, J. Dehaene, B. De Moor, and H. Verschelde, *Four qubits can be entanngled in nine different ways*, Phys. Rev. **A65** (2002) 052112 [quant-ph/0109033].

[8] O. Biham, M. A. Nielsen and T. J. Osborne, *Entanglement monotone derived from Grover's algorithm*, Phys. Rev. **A65** (2002) 062312 [quant-ph/0112097].

[9] L. K. Grover, *Quantum Mechanics helps in searching for a needle in a haystack*, Phys. Rev. Lett. **79** (1997) 325 [quant-ph/9706033].

[10] A. Karlsson and M. Bourennane, *Quantum teleportation using three-particle entanglement*, Phys. Rev. **A58** (1998) 4394.

[11] W. K. Wootters and W. H. Zurek, *A single quantum cannot be cloned*, Nature, **299** (1982) 802.

[12] H. Barnum, C. M. Caves, C. A. Fuchs, R. Jozsa, and B. Schumacher, *Noncommuting mixed states cannot be broadcast*, Phys. Rev. Lett. **76** (1996) 2818 [quant-ph/9511010].

[13] V. Bužek, V. Vedral, M. B. Plenio, P. L. Knight, and M. Hillery, *Broadcasting of entanglement via local copying*, Phys. Rev. **A55** (1997) 3327 [quant-ph/9701028].

[14] N. Gisin and S. Massar, *Optimal Quantum Cloning Machines*, Phys. Rev. Lett. **79** (1997) 2153 [quant-ph/9705046].

[15] V. Bužek and M. Hillery, *Quantum copying: Beyond the no-cloning theorem*, Phys. Rev. **A54** (1996) 1844 [quant-ph/9607018].

[16] A. K. Pati, *Assisted cloning and orthogonal complementing of an unknown state*, Phys. Rev. **A61** (2000) 022308 [quant-ph/9904068].

[17] P. Agrawal and A. Pati, *Perfect teleportation and superdense coding with W states*, Phys. Rev. **A74** (2006) 062320 [quant-ph/0610001].

[18] E. Jung, Mi-Ra Hwang, H. Kim, M. -S. Kim, D. K. Park, J. -W. Son and S. Tamaryan, *Groverian Measure in Multi-Qubit Systems*, arXiv:0709.4292 [quant-ph].

[19] L. Tamaryan, D. K. Park, and S. Tamaryan, *Analytic Expression for Geometric Measure of Three Qubit States*, Phys. **A 77** (2008) 022325 arXiv:0710.0571 [quant-ph].

[20] Y. Shimoni, D. Shapira and O. Biham, *Characterization of pure qyantum states of multiple qubits using the Groverian entanglement measure*, Phys. Rev. **A69** (2004) 062302 [quant-ph/0309062].

[21] C. H. Bennett and S. J. Wiesner, *Communication vi one- and two-particle operators on Einstein-Podolsky-Rosen states*, Phys. Rev. Lett. **69** (1992) 2881.

[22] Xin-Wei Zha and Hai-Yang Song, *Three-qubit pure-state canonical forms for Perfect Teleportation*, arXiv:0706.3326 [quant-ph].

[23] L. Tamaryan, D. K. Park, J. W. Son and S. Tamaryan, *Geometric Measure of Entanglement and Shared Quantum States*, arXiv:0803.1040 [quant-ph].